1 Number Patterns Exhibiting Geometric Growth

Many real-life situations can be represented by a pattern of numbers which exhibit **geometric growth**.

1.1 Example

In 2010, the University of Victoria in Victoria, B.C. had a problem with rabbits. It was estimated that, at the time, there were 1500 rabbits on campus.

(a) If the rabbit population increases at a rate of 20% per year, complete the table below, estimating the rabbit population in the years 2011, 2012, and 2013:

Year	2010	2011	2012	2013
Number of Rabbits				

(b) Show that the sequence representing the rabbit population is gemoetric, and find the common ratio:

(c) If term 1 in the sequence is the number of rabbits in 2010, which term represents the number of rabbits in 2025?

(d) If the population continues to grow at this rate, estimate the population of rabbits in 2025.

Note: The **geometric growth factor** is another name for the common ratio in a pattern that exhibits geometric growth.

In this example, the geometric growth factor is _____.

2 Number Patterns Exhibiting Geometric Decay

Many real-life situations can be represented by a pattern of numbers which exhibit **geometric decay**.

2.1 Example

You borrow \$10 000 from your parents to buy a car. Your parents lend you the money interest-free, as long as you make a payment at the end of each year. You've agreed that at the end of each year, you will owe 10% less than you did at the beginning of each year. This continues for 8 years, at which point you will pay the remaining balance.

(a) Complete the table below by determining how much money you owe at the end of each of the first four years.

End of Year	1	2	3	4
Amount Owing (\$)				

(b) The values form a geometric sequence. Calculate the common ratio.

(c) What is the geometric growth factor?

(d) Determine how much you owe at the end of the seventh year.

Note: Geometric decay is just a special case of geometric growth, where the common ratio is between 0 and 1!

2.2 Example

State the growth factor for each situation:

(a) The rate of inflation is increasing by 3.5% each year.

(b) The number of fish in a lake is decreasing by 2% each year.

(c) The number of rabbits in a population is doubling each year.

(d) The value of a computer decreases by one-fifth each year.

(e) The ball rebounds to $\frac{3}{4}$ of its previous height after each bounce.

2.3 Example

"A rubber ball is dropped from the top of a building 20m high. Each time the ball bounces, it bounces up to 80% of its previous height. Calculate, to the nearest cm, the height of the ball after the fifteenth bounce."

(a) Austin decides to solve the problem with a gemoetric sequence, using $t_1 = 20$.

(i) Explain why using t_{15} does not answer the problem.

(ii) Use Austin's method to solve the problem.

(b) Nicole thinks that the value of $t_1 5$ *should* be the answer to the problem - after all, it's asking about the fifteenth bounce!

(i) What will the first term of Nicole's sequence have to be?

(ii) Use Nicole's method to solve the problem.

(c) For each student, write an equation which represents the height of the ball after x bounces.

(d) Write an equation which could be solved to determine the minimum number of bounces required for the rebound height to be less than 2 cm.

(e) Use the intersection of two graphs to solve the previous equation.

3 Compound Interest

OceAnna invests \$5000 in a guaranteed investment certificate (GIC). Each year, 3% interest is added to the value of the investment at the beginning of that year. Consider the geometric sequence which represents the value of the investment at the **end of each year**.

(a) State the growth factor.

(b) Write a **product** which represents the value of the investment after one year. This will represent the first term of the geometric sequence.

(c) Write a **product** which represents the value of the investment after two years.

(d) Use the formula $t_n = ar^{n-1}$ to write an expression which could be used to evaluate the value of the investment after ten years. Simplify (but do not evaluate) this expression.

(e) State the value of the investment, to the nearest cent, after

(i) one year (ii) two years (iii) ten years

(f) Use the formula $t_n = ar^{n-1}$ to write an expression which could be used to calculate the value of the investment after n years. Show that this expression can be written as a power with an exponent of n.

(g) Suppose that an investment of P dollars earns interest of i% per year. Develop a formula fo the amount A of the investment after n years. This formula is known as the **compund interest formula**.